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DETERMINATION OF NATURAL FREQUENCIES AND STABILITY REGIONS OF AXIALLY MOVING BEAMS USING ARTIFICIAL NEURAL NETWORKS METHOD

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1. INTRODUCTION

The field of axially translating materials consists of high-speed magnetic and paper tapes, thread lines, strings, power transmission chains and belts, band-saws, fibers, beams, aerial cable tramways and pipes conveying fluid and other similar systems. Ulsoy *et al.* [1] and Wickert and Mote [2] reviewed the literature on axially moving materials. Miranker [3] studied a model for the transverse vibrations of a tape moving between a pair of pulleys using a variational procedure and derived the equations of motion for time-dependent axial velocity. Mote [4] considered the problem of an axially accelerating string with harmonic excitation at one end and made a stability analysis. Wickert [5] analyzed free non-linear vibrations of a moving beam over the sub- and super-harmonic transport speed ranges. Pakdemirli and Batan [6] considered a constant acceleration–deceleration-type motion. Zhang and Zu [7] performed modal analysis of linear prototypical serpentine belt derive system. Pakdemirli and Ulsoy [8] studied principal parametric resonances and combination resonances for an axially accelerating string. Stylianou and Tabarrok [9, 10] used finite element formulation to show the accuracy of variable-domain beam element, considered translational and rotary inertia effects of the tip mass and made a stability analysis. Pakdemirli *et al.* [11] derived the equations of motion for an axially accelerating string using Hamilton's principle and numerically investigated the stability. Öz *et al.* [12] investigated the transition from string to beam for an axially moving material. Approximate analytical expressions for the non-linear natural frequencies were given for the problem. For variable velocity profiles, stability borders were determined analytically. Euler–Bernoulli beams having different flexural stiffness values and moving with harmonically varying velocities for different end conditions are studied [13–15]. Principal parametric resonances, sum and difference type combination resonances were investigated. As a special case, the vibrations of a tensioned pipe conveying fluid with variable velocity is investigated.

Artificial neural networks (ANN) are networks of simple processing elements called neurons operating on their local data and communicating with other elements. The design of ANNs was motivated by the structure of a real brain, but the processing elements and the

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architectures used in ANN have gone far from their biological inspiration. Each neuron is connected at least with one neuron, and each connection is evaluated by a real number, called the weight coefficient, that reflects the degree of importance of the given connection in the neural network. Knowledge is stored in the form of a collection of connection strengths. ANNs are capable of self-organization and knowledge acquisition (learning). This capability allows automatic determination from the connection strengths from data containing the knowledge to be expected. Back-propagation is a time-consuming algorithm. The processing units are arranged in layers. Each ANN has an input layer, an output layer and a number of hidden layers. Propagation takes place in a feed-forward manner, from input layer to the output layers. The pattern of connectivity and the number of processing units in each layer may vary with some constraints. No communication is permitted between the processing units within a layer, but the processing units in each layer may send their output to the processing units in the higher layer [16–22]. For some other examples of ANN applications to structural mechanics, the reader is referred to references [22–31].

In this study, the transverse vibration of a Euler–Bernoulli-type axially moving beam is investigated. The beam is simply supported at both ends. Axial velocity is assumed as a harmonic function about a constant mean value. The frequency values and stability borders obtained in a previous study are trained using ANN. For new value of flexural stiffness and mean velocities, frequencies and stability borders are determined using ANN.

2. EQUATION OF MOTION AND APPROXIMATE SOLUTION

The linear dimensionless equations of motion for the travelling beam shown in Figure 1 is [5, 13]

$$(\ddot{w} + 2\dot{w}'v + w'\dot{v}) + v_f^2 w^{iv} + (\check{v}^2 - 1)w'' = 0 \tag{1}$$

and simple–simple boundary conditions are

$$w(0, t) = w(1, t) = 0, \qquad w''(0, t) = w''(1, t) = 0. \tag{2}$$

In equation (1), w is the transverse displacement, v is the axial velocity, $\ddot{w}$ is the local acceleration, $2\dot{w}'v$ is the Coriolis acceleration, $v^2 w''$ is the centripetal acceleration, and v_f^2 denotes flexural stiffness. The variable velocity is assumed as a harmonically varying function about the constant mean value

$$v = v_0 + \varepsilon v_1 \sin \Omega t, \tag{3}$$

where ε is a small parameter and εv_1 represents the amplitude of fluctuations. Ω is the velocity fluctuation frequency. The solution has been given in reference [13] by applying the method of multiple scales (a perturbation technique). The dispersion relation and support

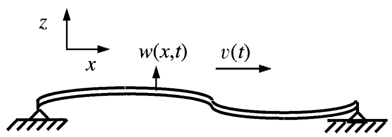

Figure 1. Schematics of an axially moving beam on simple supports.

condition obtained from the boundary conditions are [13]

$$v_f^2\beta_n^4 + (1 - v_0^2)\beta_n^2 - 2\omega_n v_0 \beta_n - \omega_n^2 = 0, \tag{4}$$

$$\begin{aligned}&[e^{i(\beta_{1n}+\beta_{2n})} + e^{i(\beta_{3n}+\beta_{4n})}](\beta_{1n}^2 - \beta_{2n}^2)(\beta_{3n}^2 - \beta_{4n}^2)\\ &\quad + [e^{i(\beta_{1n}+\beta_{3n})} + e^{i(\beta_{2n}+\beta_{4n})}](\beta_{2n}^2 - \beta_{4n}^2)(\beta_{3n}^2 - \beta_{1n}^2)\\ &\quad + [e^{i(\beta_{2n}+\beta_{3n})} + e^{i(\beta_{1n}+\beta_{4n})}](\beta_{1n}^2 - \beta_{4n}^2)(\beta_{2n}^2 - \beta_{3n}^2) = 0.\end{aligned} \tag{5}$$

These equations were solved for $v_f = 0{\cdot}1$, 0·2, 0·4, 0·6, 0·8, 1·0 in reference [13] and these values will be used in the training phase of ANN in the next section. The stability borders for principal parametric resonance case (for the velocity fluctuation frequency close to two times of any natural frequency) were given in reference [13] as

$$\Omega = 2\omega_n \mp 2\varepsilon\sqrt{k_{0R}^2 + k_{0I}^2}, \tag{6}$$

where k_{0R} and k_{0I} are the real and imaginary parts of k_0,

$$k_0 = \frac{\left\{\frac{1}{2}(\Omega - 2\omega_n)\int_0^1 \bar{Y}_n' \bar{Y}_n \,\mathrm{d}x - \mathrm{i}v_0 \int_0^1 \bar{Y}_n'' \bar{Y}_n \,\mathrm{d}x\right\}}{2\left\{\mathrm{i}\omega_n \int_0^1 \bar{Y}_n Y_n \,\mathrm{d}x + v_0 \int_0^1 \bar{Y}_n Y_n' \,\mathrm{d}x\right\}} v_1, \tag{7}$$

where $Y_n(x)$ is a spatial function; $\bar{Y}_n(x)$ is the complex conjugate. The stability borders were found and drawn for $v_f = 0{\cdot}1$, 0·2, 0·4, 0·6, 0·8, 1·0 in reference [13]. These values will be used in the training phase of the ANN method to obtain the stability borders for the new flexural stiffness value.

3. ARTIFICIAL NEURAL NETWORK APPROACH (ANN)

Artificial neural network (ANN) algorithm is applied in this section as an alternative to the conventional numerical methods. ANN systems are physical cellular systems that can acquire, store and utilize experimental knowledge. The distinguished characteristics of neural networks have played an important role in a wide variety of applications. Powerful learning algorithms self-adapt as per the requirements in a continually changing environment (adaptability property). The ability to perform tasks involving non-linear relationships and noise immunity make ANN a good candidate for classification and prediction (non-linear processing property). Finally, architectures with a large number of processing units enhanced by extensive interconnectivity provide for concurrent processing as well as parallel distributed information storage (parallel processing property). The multi-layer perceptron (MLP) has an input layer, hidden layers and an output layer. The input vector representing the pattern to be recognized is incident on the input layer and is distributed to subsequent hidden layers, and finally to the output layer via weighted connections. Each neuron in the network operates by taking the sum of its weighed inputs and passing the result through a non-linear activation function (transfer function). Generally, the sigmoid function is chosen as the non-linear activation function.

In this study a multi-layer perceptron, feed-forward and back-propagation algorithm ANN is used by supervised training. Details of the algorithm can be found in reference [29]. The ANN method is applied for two different solutions. First, the natural frequencies are calculated. Two variables for input and one variable for output values were considered in

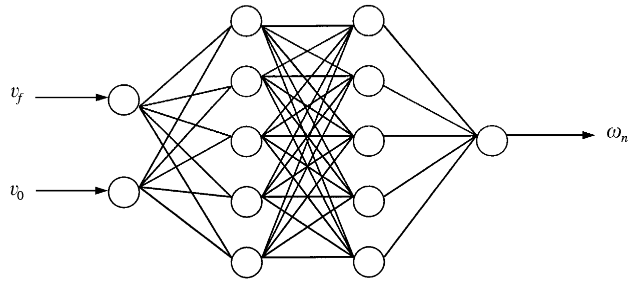

Figure 2. ANN used in determining the velocity-dependent natural frequencies.

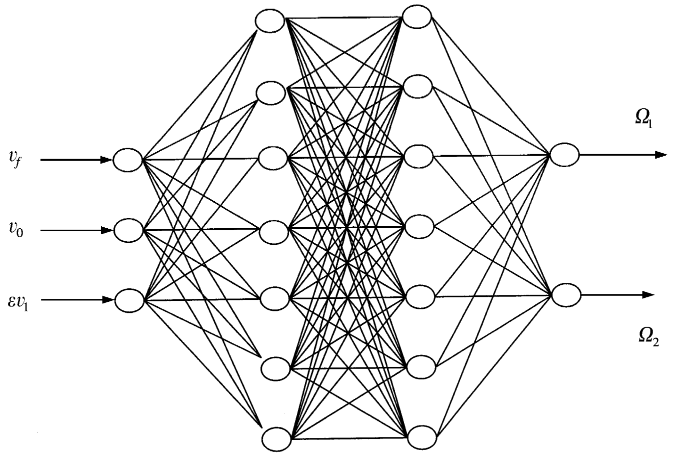

Figure 3. ANN used in determining the stability borders.

the first application. The inputs are the flexural stiffness value ($\check{v}_f$) and the mean value of axial velocity (v_0), and the output value is the natural frequency (ω_n). The training is performed separately for every natural frequency. The ANN architecture used in calculating the natural frequencies part is a 2:5:5:1 multi-layer architecture (Figure 2). The calculation of natural frequencies from the dispersion equation (4) and support condition (5) is a time consuming job. The analytically calculated values are used in ANN; then, the new frequency values for any flexural stiffness value and mean velocities can be easily obtained. In the second part, the stability borders for principal parametric resonance obtained in reference [13] using equations (6) and (7) is trained in ANN. The inputs are flexural stiffness value (v_f), mean velocity (v_0) and velocity fluctuation amplitude (εv_1) and the output is the two values of velocity fluctuation frequencies (Ω_1, Ω_2) giving stability borders. The ANN architecture used in this part is a 3:7:7:2 multi-layer architecture as shown in Figure 3. The momentum and learning rate values are taken as 0·9 and 0·7 respectively. These values are found to be optimum values by trial and error. The non-linear activation function is chosen as the sigmoid function. For both parts 100 000 iterations are performed in training the algorithm. The ANN is tested after the learning phase. The ANN used to determine the natural frequencies is tested for $v_f = 0{\cdot}3$. In Figures 4 and 5, mean axial velocity-dependent natural frequencies for $v_f = 0{\cdot}3$, calculated by analytical approach and determined using ANN are plotted for the first two modes respectively. The results are close to each other throughout the velocities except for very small values. The error in determining the

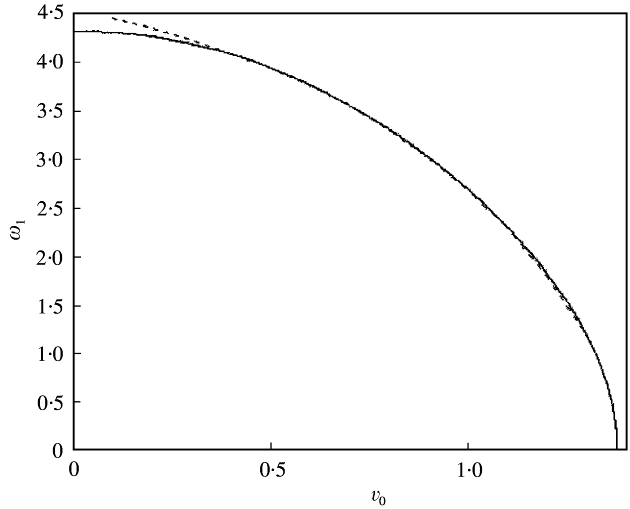

Figure 4. A comparison of analytical and ANN results for natural frequency versus mean axial velocity (——, analytical; - - - - -, ANN) in the first mode.

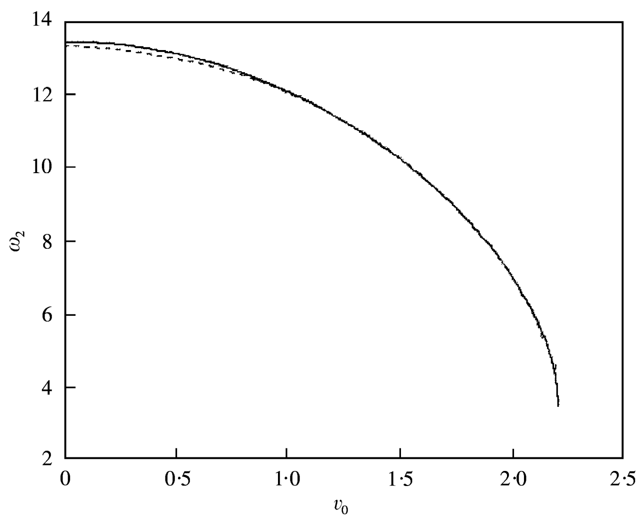

Figure 5. A comparison of analytical and ANN results for natural frequency versus mean axial velocity (——, analytical; - - - - -, ANN) in the second mode.

frequencies for small velocities arises from the normalization. That is why the minimum value for the mean velocity is chosen as 0·1. Also, sharp natural frequency changes near a critical velocity value cause error in ANN.

The ANN used to determine the stability borders is tested for $v_f = 0{\cdot}3$ for the fundamental frequency. Figures 6 and 7 show the stability borders calculated analytically and using ANN, respectively, for mean velocity and natural frequency variation for the first mode. The figures are similar to each other except again for small mean velocities and small velocity fluctuation amplitudes.

Using ANN, one can eliminate excessive analytical and numerical calculations for natural frequencies and stability borders for every flexural stiffness value. Otherwise for

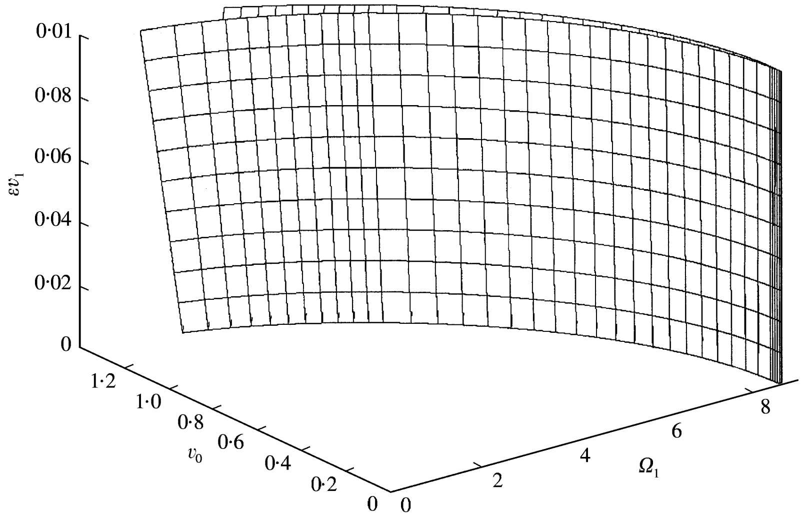

Figure 6. The borders separating the stable and unstable regions calculated by the analytical approach.

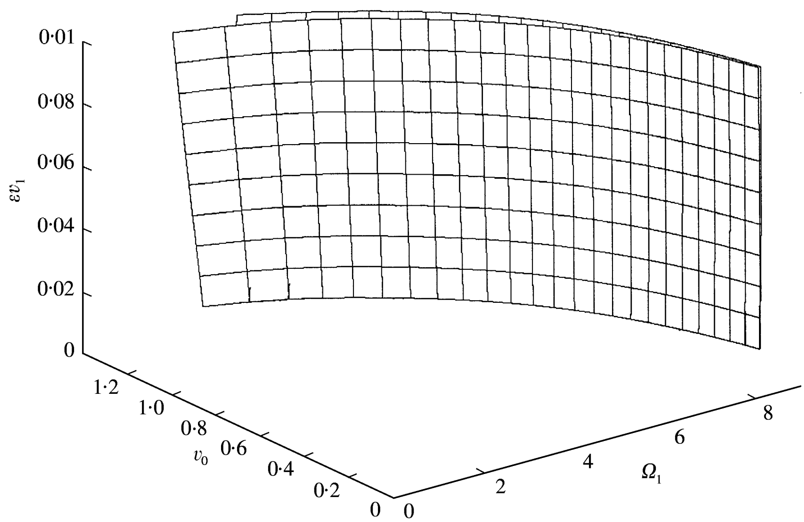

Figure 7. The borders separating the stable and unstable regions determined using ANN.

every new parameter, new frequency values and stability borders have to be calculated again. This can be a way of shortening the computing efforts.

4. CONCLUDING REMARKS

The transverse vibrations of an axially accelerating Euler–Bernoulli beam on simple supports are investigated. The beam travels with a sinusoidal function about a constant mean value. The application of artificial neural networks (ANN) in calculating natural frequencies and stability borders has been presented. The pre-calculated natural frequency values and stability borders using the method of multiple scales for different flexural stiffness coefficients are used in ANN training. The velocity-dependent natural frequencies and the stability borders are drawn using ANN. The comparison of the analytical solutions and ANN show that the ANN approach is satisfactory. With sufficient training, the ANN results will give unknown values approximately. Time consumption and expensive computer operations can be reduced with this method.

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